

SIMON FRASER UNIVERSITY
Faculty of Business Administration

FINAL EXAM

BUS 411 Fixed Income Security Analysis
Prof. Geoffrey Poitras

24-3

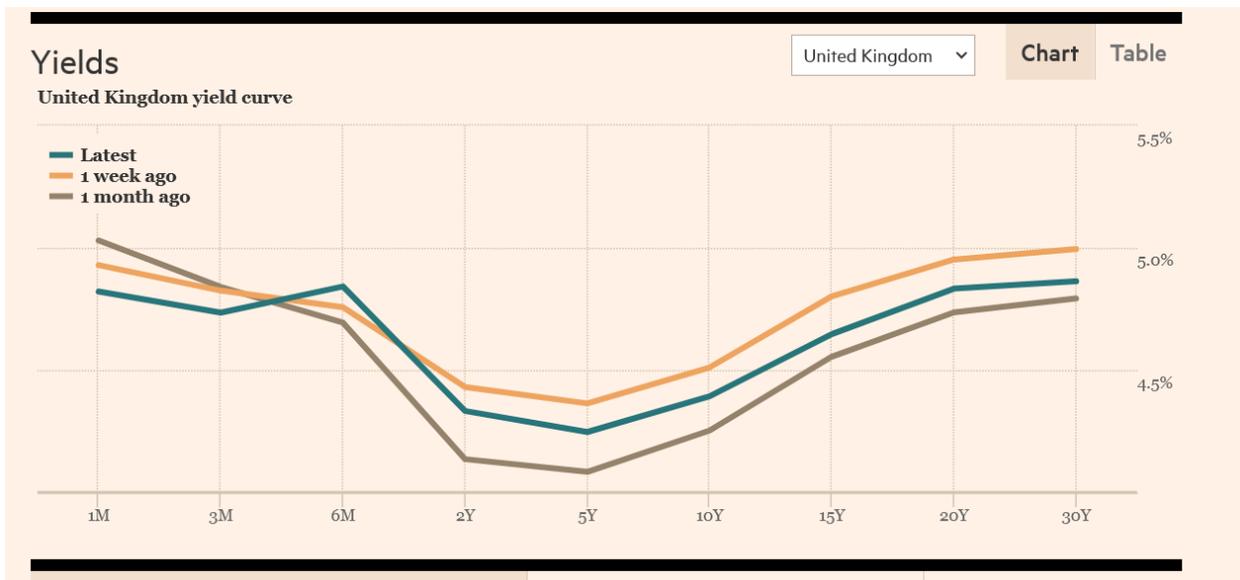
Academic Honesty: This assignment is individual work. Students are required to follow requirements of S10.01, especially Appendix A (see class web page for link).

Rules for Submitting Final Exam: Exam is due in my email (poitras9@sfu.ca) no later than Noon on Wed. Dec. 11, 2024. Late assignments will be assessed a 2% reduction per hour or part thereof, e.g., 11.5 hours late will have a 24% reduction.

DO ALL PARTS OF ALL QUESTIONS: Each question is worth 25 total points – for questions with two parts 15 points for part a) and 10 points for part b). **Where applicable, provide calculations or code used to determine answers.**

1. For the “Latest” Yield curve in the Chart below:

a) Calculate all annual implied zero coupon interest rates (z_t yearly) for the 1 to 10 year yield curve. With these z_t calculate a set of one year ahead implied forward rates (${}_t f_{t+1}$)



(Hint: Assume that the chart is for annual coupon par bond yields and that the 1 year yield is for a zero coupon.)

ii) Using the par bond yield curve from Question #1a), construct a portfolio of a 2 year + 30 year bond that has the same modified duration as the ten year bond. What are the convexities and ‘time value’ for the 2 + 30 year portfolio and the 10 year bond? Calculate and explain the ‘time value-convexity’ trade-off for this yield curve.

2.a) i) On Nov. 22, 2024 the Chemtrade Logistics 7% debenture maturing June 30, 2027 was selling for \$106.50 (all-in, not flat), what is the promised yield to maturity for this bond? On the same day, the Slate Office REIT 7.5% debenture maturing Dec. 31, 2027 was selling for \$41.00 (all-in, not flat), what is the yield to maturity for this bond? How would the answers change if the bond indentures provide for conversion into units?

ii) You have just taken up a variable rate mortgage with a 25 year amortization at 5.15% (prime rate minus 60 bp). Assume that in one year the prime rate falls by 80 basis points. Calculate the amortization period remaining on the mortgage after the fall in the prime rates. (Provide the program/calculations used to arrive at the solution). If at the same time the prime rate falls, you decide to take up the opportunity to make a principal payment of 10%, what will the remaining amortization be after taking account of the principal reduction and rate reduction?

iii) Halley (1698) poses the following problem: An annuity of £20 being in possession for the term of 21 years, and for £40 paid down it can be prolonged for 10 years more to 31 years; what is the rate of interest required? (Hint: Evaluate the interest rate associated with paying £40 today for a cash flow of £20 that will last for 10 years after the 21 year annuity reaches maturity.)

2.b) Adam Smith provides the following description of the Million £ Loan:

During the reigns of king William and queen Anne, large sums were frequently borrowed upon annuities for terms of years, which were sometimes longer and sometimes shorter. In 1693, an act was passed for borrowing one million upon an annuity of fourteen per cent., or of 140,000 l. a year, for sixteen years. In 1691, an act was passed for borrowing a million upon annuities for lives, upon terms which in the present times would appear very advantageous. But the subscription was not filled up. In the following year the deficiency was made good by borrowing upon annuities for lives at fourteen per cent., or at little more than seven years' purchase. In 1695, the persons who had purchased those annuities were allowed to exchange them for others of ninety-six years, upon paying into the Exchequer sixty-three pounds in the hundred; that is, the difference between fourteen per cent. for life, and fourteen per cent. for ninety-six years, was sold for sixty-three pounds, or for four and a half years' purchase. Such was the supposed instability of government, that even these terms procured few purchasers. In the reign of queen Anne, money was upon different occasions borrowed both upon annuities for lives, and upon annuities for terms of thirty-two, of eighty-nine, of ninety-eight, and of ninety-nine years.

i) Assuming arithmetically declining survival rates (uniform death rates), a maximum possible age of 80 and a market interest rate of 6.5%, what is the 'breakeven age' for a purchaser of the million £ loan life annuity.

ii) Explain how Smith arrives at the solution of 4.5 years purchase for the exchange of the life annuity for the term annuity of 96 years at the rate of £ 63 of term annuity for £100 of life annuity.

iii) Assuming arithmetically declining survival rates (uniform death rates) and a maximum possible age of 84, if the life annuity nominee is 15 years old, what is the 'true yield' for the life annuity?

3. Using the CME price quotes in the APPENDIX solve for the 3 and 6 month foreign interest rates implied by Covered Interest Parity using the SOFR futures and CME currency futures for the C\$ and €. (Hint: See the worked example on the class webpage; use the nearby contract for the spot price; final answers to be expressed as **annualized** interest rates.)

4. (Show Calculations, attach spreadsheet or mathematica code used to solve the problem)

The Canada Pension Plan (Canada Pension Plan, RSC 1985, c. C-8) is a complicated defined benefit plan that has several provisions, such as adjustments for 'drop-out years', that impact the required number of contribution years required to receive the maximum pension payment. In addition, the plan allows for pension payments to be started at any time between age 60 and age 70, with appropriate adjustment in the payment amount. For a female age 65 that is eligible for 2 years of 'drop-out provisions' that reduce the number of maximum contribution years to receive the full benefit, 37 qualifying years of maximum contributions is required for the full benefit of \$16,375.20 per year (\$1,364.60/mo.).

Using the maximum contribution amounts from the following Table and assuming investment returns applicable for each of the following intervals:

1987-1990	10.5%
1991-2000	7.5%
2001-2010	4.5%
2011-2023	3.25%

i) For both the employee and self-employed, calculate the amount that would have been earned (from 1984 to 2023) if the CPP contributions given in the Table below had been invested instead of being paid into CPP. (Hint: The calculation involves starting from 1987 and accumulating investment in a fund to arrive at a final total in 2023.)

Using the calculation from a) for both an employee and a self-employed individual that has made the maximum contributions detailed in the Table, solve for the following:

ii) Assuming arithmetically declining survival rates and a maximum possible age of 95, calculate the implied interest for this individual electing to receive the maximum CPP pension payment at age 65 of \$16,375.20 per year (\$1,364.60/mo.). (Hint: This is the same type of calculation as that for Assignment #1, 2b)

iii) For both the employee and self-employed, if the individual opts to defer taking the CPP until age 70 when the annual payment would be \$1937.33/month; \$23,247.96 annual (no further CPP contributions are required or made after 2023), calculate the implied interest rate assuming arithmetically declining survival rates and a maximum possible age of 95.

iv) At a current interest rate of 3.5% what is the breakeven age at which deferring CPP until age 70 has the same present value as taking CPP at age 65. (Hint: This involves using the cash flows from b) and c) above and doing a expected present value calculation for age 65.) How does your answer change if the interest rate is 5.5%?

Table: Maximum Canada Pension Plan Contributions, 1984-2022

	<u>Employee Contribution</u>	<u>Self-Employment Contribution</u>
1987	\$444.60	\$889.20
1988	\$478.00	\$956.00
1989	\$525.00	\$1050.00
1990	\$574.20	\$1148.40
1991	\$632.50	\$1263.00
1992	\$696.00	\$1392.00
1993	\$752.50	\$1505.00
1994	\$806.00	\$1612.00
1995	\$850.50	\$1701.00
1996	\$925.82	\$1851.62
1997	\$993.22	\$1996.44
1998	\$1,068.80	\$2,137.6
1999	\$1,186.50	\$2,373.00
2000	\$1,329.90	\$2,659.80
2001	\$1,496.40	\$2,992.80
2002	\$1,673.20	\$3,346.40
2003	\$1,801.80	\$3603.60
2004	\$1,831.50	\$3663.00
2005	\$1,861.20	\$3,722.40
2006	\$1,910.70	\$3821.40
2007	\$1,989.90	\$3979.80
2008	\$2,049.30	\$4,098.60
2009	\$2,118.60	\$4,337.20
2010	\$2,163.15	\$4,236.20
2011	\$2,217.60	\$4,435.20
2012	\$2,306.70	\$4,613.40
2013	\$2,356.20	\$4,712.40
2014	\$2,425.50	\$4,851.00
2015	\$2,479.95	\$4,959.90
2016	\$2,544.30	\$5,088.60
2017	\$2,564.10	\$5,128.20
2018	\$2,593.80	\$5,187.60
2019	\$2,748.05	\$5,497.80
2020	\$2,898.00	\$5,796.00
2021	\$3,166.45	\$6,332.90
2022	\$3,499.80	\$6,999.60
2023	\$3,754.45	\$7,508.90

<https://www.canada.ca/en/revenue-agency/services/tax/businesses/topics/payroll/payroll-deductions-contributions/canada-pension-plan-cpp/cpp-contribution-rates-maximums-exemptions.html>

APPENDIX FOR QUESTION #3 Quotes for Nov. 22, 2024

CANADIAN DOLLAR FUTURES - SETTLEMENTS

TRADE DATE Friday, 22 Nov 2024 ▾

Last Updated 22 Nov 2024 11:55:00 PM CT

ESTIMATED VOLUME TOTALS 83,436

PRIOR DAY OPEN INTEREST TOTALS 344,941

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
DEC 24	.71630	.71695	.71375	.71575	-.00090	.71580	81,350	332,355
JAN 25	.71735	.71770B	.71460A	.71680B	-.00090	.71660	92	137
FEB 25	.71580	.71860B	.71550A	.71735A	-.00090	.71750	35	14
MAR 25	.71860	.71940B	.71620A	.71820B	-.00090	.71825	1,930	8,820
APR 25	-	-	-	-	-.00100	.71885	0	0
JUN 25	.71975	.72060	.71975	.72060B	-.00090	.72050	10	3,241
SEP 25	.72290	.72290	.72235	.72235A	-.00090	.72260	18	307
DEC 25	.72380	.72380	.72275A	.72380B	-.00100	.72435	1	61

EURO FX FUTURES - SETTLEMENTS

TRADE DATE Friday, 22 Nov 2024 ▾

Last Updated 22 Nov 2024 11:55:00 PM CT

ESTIMATED VOLUME TOTALS 334,121

PRIOR DAY OPEN INTEREST TOTALS 663,298

MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
DEC 24	1.048400	1.050850	1.034050	1.042800	-.007400	1.042000	321,696	637,367
JAN 25	1.049750	1.052200	1.036350	1.043950A	-.007350	1.043500	660	4,398
FEB 25	1.050800	1.053950	1.038450	1.045650A	-.007300	1.045300	513	185
MAR 25	1.053300	1.055450	1.039200A	1.047850	-.007200	1.047000	11,047	15,719
APR 25	1.053100	1.053100	1.047500A	1.053100	-.007050	1.048800	2	0
JUN 25	1.058650	1.058650	1.050650	1.053600A	-.006850	1.052950	34	2,366
SEP 25	1.063850	1.065250	1.057600	1.060000B	-.006400	1.059250	156	2,763
DEC 25	1.064850	1.072600B	1.063900	1.064900B	-.006050	1.065500	13	477

THREE-MONTH SOFR FUTURES - SETTLEMENTS

TRADE DATE Friday, 22 Nov 2024 ▾

Last Updated 22 Nov 2024 11:55:00 PM CT

ESTIMATED VOLUME TOTALS		3,944,961		PRIOR DAY OPEN INTEREST TOTALS		10,990,260		
MONTH	OPEN	HIGH	LOW	LAST	CHANGE	SETTLE	EST. VOLUME	PRIOR DAY OI
SEP 24	95.2375	95.2375	95.2350	95.2350	.0000	95.2350	23,956	1,259,920
OCT 24	95.3425	95.3475	95.3375	95.3375B	.0000	95.3450	164	5,802
NOV 24	-	95.4750B	95.4650A	95.4650A	.0000	95.4700	0	1,473
DEC 24	95.5250	95.5400	95.5150	95.5225	.0000	95.5250	631,327	1,287,043
JAN 25	95.6000	95.6100B	95.5850	95.5950A	.0000	95.6000	55	997
FEB 25	95.6850	95.6850	95.6650A	95.6850A	-.0050	95.6750	2	116
MAR 25	95.7300	95.7550	95.7150	95.7200	-.0100	95.7250	408,389	1,084,164
APR 25	95.7750	95.8000B	95.7550	95.7700	-.0100	95.7800	216	283
MAY 25	-	95.8600B	95.8400A	95.8600B	-.0100	95.8400	0	0
JUN 25	95.8950	95.9250	95.8600	95.8650	-.0300	95.8700	352,353	998,154

BONUS (5 points)

Witt (1613) poses the following problem: A oweth to B £1200 to be paid in 6 years, in 12 equal payments, viz. at the end of each halfe year £100. They agree to cleare this debt in 3 years, in 6 equall payments, viz. at the end of each halfe year, one payment. The Question is, what each payment ought to be, reckoning interest after the rate of 10 per cent per Ann. and int. upon int.?